Measuring the widths of fibre optics by lateral illumination

22862

Abstract

Light from a He-Ne laser is shone through the side of a fibre optic sample, which, according to Babinet's principle, should produce a similar diffraction pattern to a slit or wire of the same width. By producing a calibration curve of diffraction pattern periodicity and known wire diameters, the width of the fibres could be determined from their diffraction pattern to within 1% of their literature value, consistent with the widely accepted manufacturing uncertainty. The technique could be used to simultaneously measure the core, fibre and plastic coating diameters.

Raw data and scripts used are accessible at http://people.bath.ac.uk/ew639/PR4.html

1 Introduction

Fibre optics are produced by drawing molten glass into fibres. The diameter of the optical fibre which is a significant factor in a fibre's transmission properties is continuously measured by a control system which regulates the drawing process [1].

Measuring the deviation of light through fibre is a non-destructive technique which can be applied to any production method to measure the diameter of the fibres during their production [2]. Typically, measurements of fibre diameters can be made down to ± 1 μm [1] (i.e $\approx 1\%$ of the diameter of fibres examined in the experiment). A typical fibre optic has an optically dense core, surrounded by a glass cladding and then a plastic coating.

This report focuses on a diffraction based method where fibre optic strands are illuminated side-on by a He-Ne laser. By making measurements of the forward-scattering diffraction pattern produced the diameter of the fibre was determined.

Light incident on a single slit, with a width comparable to the wavelength of the light, will be diffracted. Far away from the diffraction object the angular irradiance diffraction pattern will follow a sinc^2 distribution [3], given by,

$$I(\vartheta) = I(0)\left(\frac{\sin\beta}{\beta}\right)^2, \tag{1}$$

where I is the intensity at the angle ϑ from the central maxima. The variable β is given by,

$$\beta = \frac{\pi d}{\lambda}\sin\vartheta, \tag{2}$$

where d is the width of the slit and λ is the wavelength of the laser light.

When a minima occurs in the diffraction pattern,

$$\begin{aligned}\sin\beta &= 0,\\ \beta &= n\lambda,\end{aligned} \tag{3}$$

so,

$$\begin{aligned}\frac{\pi d}{\lambda}\sin\vartheta &= n\pi,\\ n\lambda &= d\sin\vartheta,\end{aligned} \tag{4}$$

which allows the width of the slit to be determined from angles between the central peak and diffraction minima.

According to Babinet's principle, the diffraction pattern produced by complementary objects should be "precisely equal in magnitude" [3]. That is, a wire should produce the same sinc^2 distribution, given in equation (1), as a slit of the same width.

Compared to a wire, a fibre optic is more complicated. Because the fibre is transparent, unlike a wire, it will not act simply to block the light. The diffraction pattern will contain diffracted, reflected and transmitted rays which greatly increases the complexity of the diffraction pattern, not least when a fibre with a core is considered. That said, a fibre can be approximated as a thin ribbon which is insulated against the electric field of light [4]. Approximating the fibre as a thin dielectric ribbon reduces the "quite complex" ray tracing solution which is accurate to within a few percent despite the large phase distortion due to

the core [1][4]. Under this approximation Babinet's principle would once again cause the diffraction pattern produced by a fibre optic to match that of a single slit described by equation (1).

When fibres with a core are analysed there exists an angle ϑ_c at which an additional periodicity in the diffraction pattern occurs due to interference between light passing though outer layers and light suffering a time delay due to passing through the core. For small angles, ϑ_c is given by,

$$\vartheta_c \approx 2(n_1 - 1)\frac{d_2}{d_1}, \tag{5}$$

where d_2 is the diameter of the core, d_1 the diameter of the cladding and the refractive index of the core (n_2) is much greater than that of the cladding (n_1) [4]. Not only could equation (5) be applied to the core and its cladding but it could also be applied to the fibre and its plastic coating. Based on this theory, the experiment tested whether it was possible to measure the diameters of different sections of the wire in one reading.

2 Method

An apparatus was designed which directed a laser beam at the side of a wire/fibre/slit and measured the diffraction pattern produced by the sample using a linear CCD (charge couple device) connected to a computer.

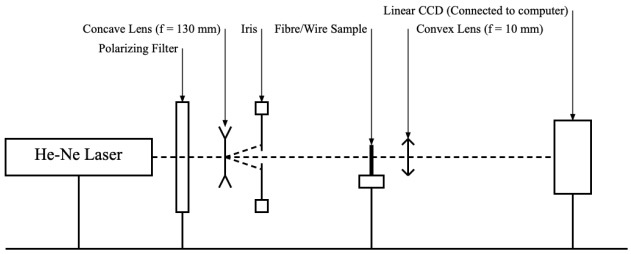


Figure 1: Diagram of apparatus. The whole setup was mounted on an optical bench. The focal lengths have been stated, although different lenses could be used to change the number of minima observed.

To reduce noise and improve the clarity of the diffraction pattern, a beam expander was used to spread the laser beam radially outwards, an iris blocked all light that did not originate from the central part of the laser beam. A polarising filter allowed the intensity of the naturally polarised He-Ne laser to be adjusted. Wire and fibre samples were mounted on a rotatable stand so that the sample was always at 90° to the linear CCD, ensuring that the diffraction pattern was parallel and centred on the CCD. A convex lens brought the diffraction pattern produced by the sample into focus on the CCD, which was placed at the focal point of the lens. The convex lens also ensured all patterns produced on the CCD were in the far-field regime - so that a Fraunhofer diffraction pattern (described by equation (1)) was produced [4].

Different powered lenses could be used when producing the diffraction pattern. The values used here were chosen so that for all the diameters of wire tested the same lenses' could be used, avoiding the need to move any of the components or recenter the laser beam.

2.1 Babinet's Principle

In order for the width of a wire to be measured, it had to be shown that Babinet's statement, that the diffraction patterns produced by complementary objects are equivalent, was true. Initially a single slit of known width was illuminated by the laser and the diffraction pattern recorded. The slit was then replaced by a wire, of the same width and, again, the diffraction pattern recorded. The diffraction pattern of a fibre with a similar width was also compared to the diffraction patterns of the slit and wire.

2.2 Wire Diffraction

Having shown that a wire produces the same diffraction pattern as a slit of the same width and that the angle is related to the wire width by equation (4), a calibration curve relating wire thickness, measured by a micrometer, and the angle between minima was produced. Under the small angle approximation ($\sin\vartheta \approx \vartheta$), for adjacent minima, equation (4) can be rearranged into the form $y = mx + c$,

$$\theta = \lambda\frac{1}{d}, \tag{6}$$

where θ is the angle between adjacent minima or pattern periodicity. Because equation (6) is in a linear form, a gradient of λ and y-intercept of 0 would be expected.

The produced calibration curve was used to convert from values of θ obtained into the diameter of the fibre. A wide range of wire diameters, ranging from 0.08 mm to 1.6 mm, were used to produce this calibration curve. A Fourier transform technique was used to extract the minima spacing's from the diffraction patterns (see figure 3).

2.3 Fibre Diffraction

Instead of wires, diffraction patterns for fibres were then measured. Altogether the diffraction pattern for five fibres were recorded. A recording was taken for each wire with the plastic coating stripped from the fibre and with the plastic coating present.

3 Results

The intensity distribution for a single slit and wire, both with a width of 160 μm, are plotted in figure 2.

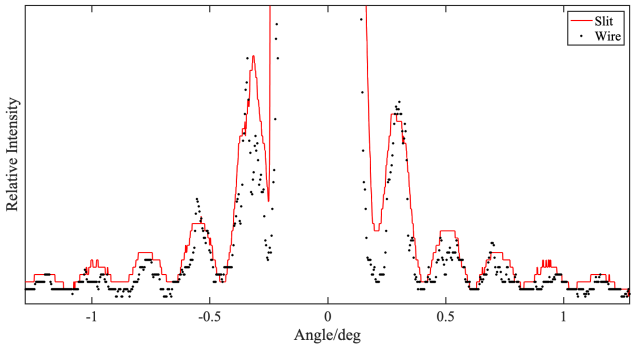
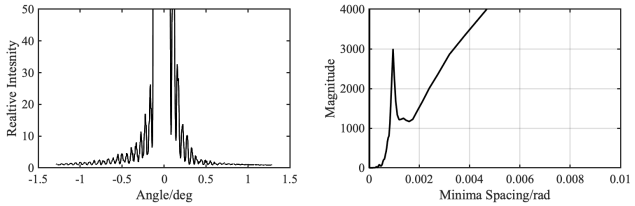


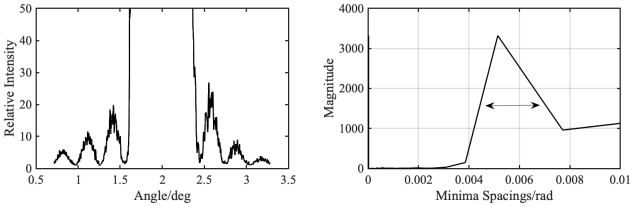
Figure 2: Intensity profiled for a wire and slit both with a width of 160 μm showing that the intensity distributions are similar.

It is clear from the graph that both the slit and wire produced the same sinc^2 intensity distribution. Using the curve fitting toolbox in MATLAB, the diameter of the wire according to equation (1) was measured to be $d_{\text{Wire}} = 164 \pm 5$ μm, and the slit to be $d_{\text{Slit}} = 162 \pm 3$ μm. Both these measurements are consistent with the 160 μm printed on the slit and ribbon. The same shape of pattern was also observed with fibres. In total six slits and ribbons had their diffraction patterns compared and in all cases the diffraction patterns showed the same distribution. We can conclude that Babinet's statement is true.

Equation (6) implies that that the minima separation is proportional to the diameter of the wire causing the diffraction. In MATLAB a function was created which found the "spectrum" of minima separations in the intensity distribution using a fast Fourier transform (FFT). An example of the diffraction pattern and its minima spacings is shown in figures 3a and 3b.



(a) 1.6 mm wire



(b) 0.08 mm wire

Figure 3: Diffraction pattern and minima spacing spectrum from fibres 1.6 mm and 0.8 mm wide. Fewer minima gave a spectrum with a much broader peak

Peaks were present in the spectrum at values of minima separation present in the diffraction pattern, with the full width half maximum (FWHM) of the peak as the uncertainty. For each of the wires, the minima separation and its uncertainty were found by using this function. The distance between minima for a selection of wires was plotted against the thickness of the wire as measured by a micrometer in figure (4).

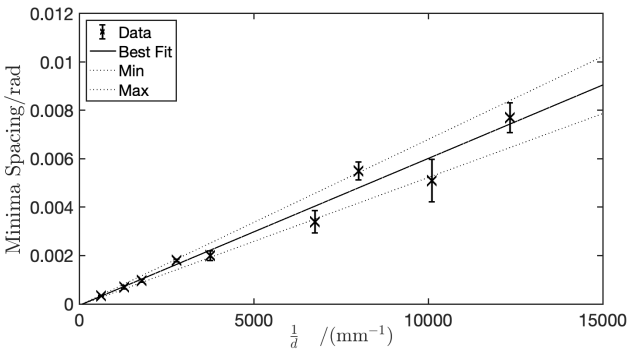


Figure 4: Calibration curve relating the distance between minima and the wire thickness.

The line of best fit connecting the points, had the equation,

$$\theta = (607 \pm 70)\text{nm}\frac{1}{d} + (-0.4 \pm 2.0) \times 10^{-5}\text{rad}, \tag{7}$$

with the uncertainties determined by the curve fitting toolbox provided by MATLAB. Equation (7) was rearranged to give a value of d, so that by measuring the minima spacing in a diffraction pattern, the width of the fibre producing the pattern could be determined.

Using the same FFT as the wires to find the minima spacing, but now plotting d according to the rearranged form of equation (7), the diameters of five optical fibres were found. For each fibre, a diffraction pattern with the plastic coating removed and present was taken. Peaks in the magnitude occur corresponding to a minima spacing present in the diffraction pattern (see section 4). A typical fibre produced a graph shown in figure 5.

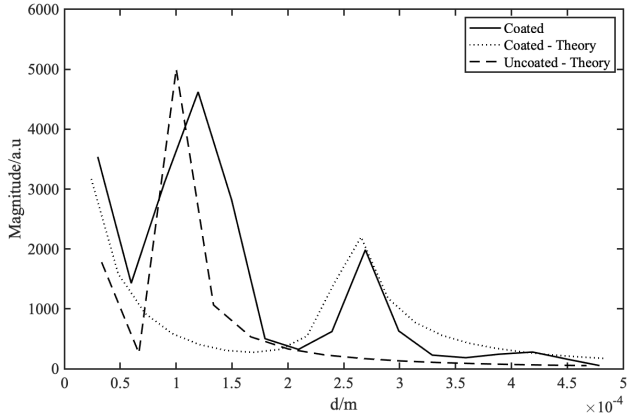


Figure 5: Diameters inside a fibre causing diffraction. Peaks occur at widths of the fibre. For clarity only the coated fibre is plotted because an uncoated fibre had the same distribution, but missing the inner peak.

For an uncoated wire only one peak was found in the spectrum, as there was only one width causing diffraction. In the case of the coated wire, an extra peak was found in the spectrum due to diffraction off of the coating and fibre (see equation (5)). Within the experimental uncertainty, the position of the peaks was consistent with the peak of a perfect theoretical intensity distribution.

From the position of the peaks the five fibres were found to have diameters as shown in table 1.

Table 1: Diameters of coated and uncoated errors. The % column's contain the percentage difference between the fibre diameter measured by a micrometer and through diffraction.

	d_{coated} (μm)	%	d_{fibre} (μm)	%
1	232	0.74	149	0.47
2	266	0.87	99.0	0.10
3	252	0.88	198	3.13
4	308	0.87	249	0.93
5	256	0.54	98.0	1.60

The column containing a % was calculated using the equation,

$$\%\text{Error} = \frac{d_{\text{diffraction}} - d_{\text{micrometer}}}{d_{\text{micrometer}}} \times 100. \quad (8)$$

4 Discussion

From equation (6), the gradient of the line of best fit should have the value of a HeNe laser - 632.991 nm [5]. Using the weighted linear regression, a gradient of (607 ± 70) nm was measured, a value which contained the literature value. The curve fit used in figure 4 was completed using the MATLAB curve fitting toolbox. Each data point was assigned a weight from 0 to 1 based on the size of the error bar, these weights were used in calculating the linear regression. The weights were calculated by dividing the minimum error by the size of the error for which the weight was calculated, giving a maximum weight of 1 for the smallest error bar and a range of values between 0 and 1 for the remaining points.

Weighting the data based on its error bar size gave a better fit to the data. Since $\theta \propto \frac{1}{d}$, thin wires produced very wide minima. Consequently, there were fewer minima recorded, with proportionally higher noise, so the minima spacing determined from the diffraction pattern had a larger error, see figure 3. The thick wire with more minima gave very sharp peak (figure 3a) whereas the thinner wire which had a much wider diffraction pattern had a very wide peak (figure 3b) with a large uncertainty.

The main limitation which caused the majority of the experiment's uncertainty was the resolution of the CCD. In the worst case scenario (thinnest optical fibre) there was only $1\frac{1}{2}$ minima either side of the saturated centre. The sensor was 2048 pixels wide, however approximately half of these pixels were lost to the centre where the undiffracted laser beam saturated the sensor. Because of the limitations in sampling frequency, diameters could only be measured in some cases down to only 50 μm intervals, $\approx$ 20% of the coated diameter or even 50% of the fibre diameter (see figure 5). Had more minima been measured in the diffraction pattern, a sharper peak with less uncertainty could have been produced, allowing the fibre diameter to be determined more precisely. The consequences of this precision are evident in table 1. The percentage error of the thin fibre, in general, is much greater than that of the diameter of the plastic coating.

As previously stated the diffraction pattern had

to be orthogonal to the linear sensor, or the intensity would quickly diminish as the pattern was no longer on the sensor. A solution to this problem was to keep minima either side of the saturated centre, to ensure that the amplitudes on either side were symmetric - thus ensuring the pattern was not lopsided. A potential improvement would be to ensure the orthogonality as mentioned above, then laterally move the CCD so that more minima were measured, which would allow the uncertainty in minima spacing's to be reduced. Although the measured angle would no longer reflect its true value, when calculating the minima spacing only the relative values of the angles were important as the period was being measured. Indeed, this was observed when analysing the data. Regardless of the data processed (excluding the saturated central section), the same spectrum was always obtained with the exception of small variations in the magnitude of spacing's. As much data (including data either side of the central saturated region) was used to reduce the uncertainties previously discussed.

In practice the diameters calculated in table 1 reflected the true diameters with uncertainties of a similar size the natural fibre deviation [1][4], rather than the peak width from figure 5. Again, the discrepancy between uncertainties could be accredited to the lack of minima measured in some results. As shown in figure 5, when the spectrum was calculated for even "perfect" sinc^2 results (provided by the measurement software) for a given fibre the width of peaks still had a finite thickness and therefore uncertainty despite being a perfect distribution. The fewer results used, the greater this width was (in both theory and fibre measurements) due to the nature of a discrete Fourier transform (DFTF) and the low sampling frequency. Since the discrepancy between the values of the micrometer and diffraction technique was less than 1%, it is evident that the peak centre reflects more precisely the diameter of the wire than the width of the peak.

Alternatively, different powered lenses could be used to control the number of minima produced. The lenses used here were chosen due to the wide range in diameters of the fibres/wires measured. The focal lengths selected allowed minima spacing's for 0.08 mm wire up to 1.6 mm to be measured. Changing the lens would have meant readjusting and realigning the distances and positions of most of the components, a process which would introduce more sources of error. In future experiments, the focal lengths of the lenses could be fine tuned to give an appropriate number of minima on the sensor, however in this experiment a range of different diameters was targeted over precision.

In some cases the core diameter was measured for thicker wires. Core diameter varies greatly depending on the type of fibre, but is generally in the range of 10 μm to 100 μm [4]. The fibre optic's refractive index is graded and continuously varies through the fibre [2]. As such any core diameters would be subject to a) a large uncertainty in diameter due to continuously varying nature of the core and b) an even greater uncertainty from wide minima due to insufficient measurement resolution as previously mentioned. As theorised, at angles greater than ϑ_c, the diffraction pattern contained additional periodicity because of the extra layers causing interference effects.

Two fibres were found to contain three peaks, corresponding to the core, fibre diameter and plastic coating diameter. The extra periodicity appeared as additional peaks in the spectrum of d, see figure 6. In all but two of the cases, the core diameter was so narrow that the additional periodicity at ϑ_c was outside the range of measurements.

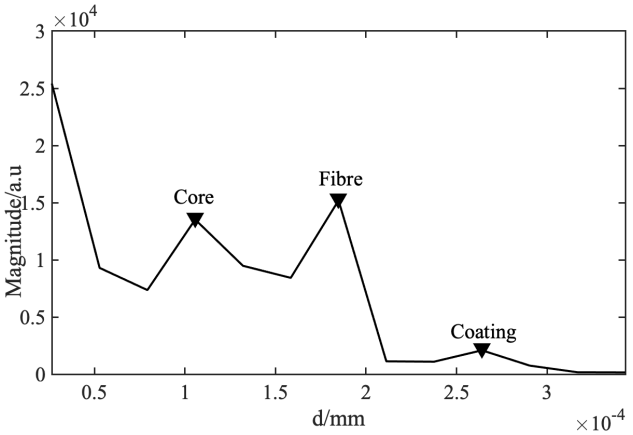


Figure 6: Contribution to of different fibre layers to the spectrum of wire showing 3 peaks; one due to the optically dense core; one due to the diameter of the glass fibre; and one peak from the plastic coating.

Figure 7 shows the measured diffraction pattern and the theoretical distribution off of the core. Extra periodicity (theory not plotted to maintain clarity) can be seen which was causing the additional peaks corresponding to the fibre and coating width, seen in figure 6.

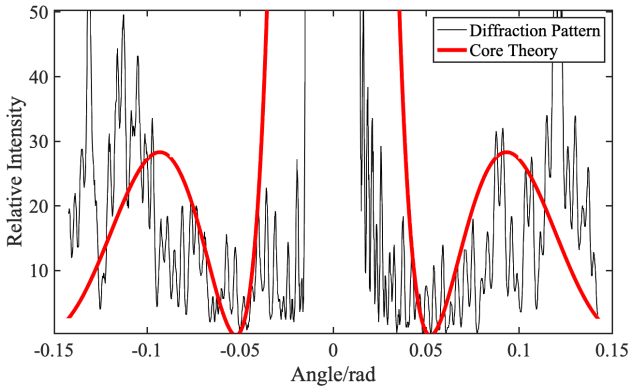


Figure 7: Diffraction pattern from fibre containing diffraction from the optically dense core with the theory for the core superimposed.

It has therefore been shown that multiple dimensions within the wire can be determined simultaneously from several values of periodicity measured in a single diffraction pattern.

Another point of interest which may wish to be explored in further experiments was the spectrum of d when d was outside of the bounds of the fibre width. Figure 5 only shows d on the scale of the fibre and coating width, however there were peaks, although two orders of magnitude lower, beyond these values. When the spectrum of both the coated and uncoated fibre's were graphed against one another these spectra were very similar, both sharing the positions of the peaks, shapes and relative amplitudes of the peaks very closely. That said, the peaks were normally closer to one another than the uncertainty in their position. Again, the peaks were severely limited in their resolution and the spectra were not identical.

It is possible that there would be extra peaks in the spectrum for the coated fibre. The peaks were likely due to reflections inside the fibre. As the light has been reflected, it may have a phase difference which causes a peak to appear at a wider diameter than the fibre width. In this case, one could expect extra peaks in the spectrum of the coated fibre as seen in figure 5 to be dependent on the refractive index of the plastic. In general this wasn't the case, however there is potential that the refractive indices of the coating didn't fulfil the criteria to produce extra peaks outside of the fibre diameter, since we know additional periodicity depends on refractive index from equation (5).

The spectrum was different for different fibres so it is unlikely there was an error in the FFT calculation. Moreover, if the spectrum was due to noise or uncertainty in measurements the similarities between the coated and uncoated fibres wouldn't be seen. Due to the low resolution and high uncertainty, no conclusion was drawn as to the cause of these similar spectra, however future experiments with improved accuracy may give enlightenment.

5 Conclusion

The diffraction technique was successfully used to measure fibres down to within 1% uncertainty of their coated value and 2% of their uncoated value. Given that natural fibre deviations are approximately 1% [1][4], despite the limited resolution of the equipment the measurement uncertainty was of a similar size to that of the fibre deviations. By using a Fourier transform it was possible to measure the fibre and plastic coating, and in some cases even core, diameters simultaneously without stripping the coating - a tricky process resulting in lots of broken fibres.

References

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